

FYBSc Semester I

Subject: Mathematics

(Algebra I & Calculus I)

Question Bank

1. Let $f: R \rightarrow R$ be given by $f(x) = 2x + 4$ and $g: R \rightarrow R$ be given by $g(x) = \sin(4x^2)$.
Find $f \circ g$, $g \circ f$, $f \circ g \circ f$, $g \circ f \circ g$. Further show that $f \circ g \neq g \circ f$.
2. $f(x) = x + \frac{1}{x-1}$, $g(x) = \sin\left(\frac{1}{x}\right)$, find $f \circ g$, $g \circ f$.
3. Define the following relations: (a) Reflexive, (b) Symmetric, (c) Transitive (d) Equivalence Relation.
4. Let X denote the set of human beings. Check whether the following is an equivalence relation or not.
 - (i) $R = \{(a, b): a \text{ \& b work in the same office}\}$
 - (ii) $R = \{(a, b): a \text{ is the son of b}\}$
 - (iii) $R = \{(a, b): a \text{ is shorter than b}\}$
 - (iv) $R = \{(a, b): a \text{ \& b live in adjacent houses.}\}$
5. State all properties of the set of natural numbers.
6. State the 1st Principle of Finite Induction.
7. Show that $n! > n^2$, $\forall n \geq 4$, using the 1st principle of induction.
8. Show that $1 + 3 + 5 + \dots + (2n - 1) = n^2$, $\forall n \in N$, using the 1st principle of induction.
9. Show that $1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$, $\forall n \in N$.
10. State Division Algorithm for both integers and polynomials.
11. Find the GCD of the following integers a and b & express GCD as a linear combination of a and b
 - (i) $a = 46$, $b = 2754$,
 - (ii) $a = 453$, $b = 2763$
12. Let $a, b, c, d, x, y \in Z$ & $n \in N$, Show that
 - (i) $a \equiv a \pmod{n}$,
 - (ii) $a \equiv b \pmod{n} \rightarrow b \equiv a \pmod{n}$,
 - (iii) $a \equiv b \pmod{n} \text{ \& } b \equiv c \pmod{n} \rightarrow a \equiv c \pmod{n}$

13. Find the last digit of 7^{65} .

14. Define Euler's phi function $\phi(n)$ & find $\phi(2178)$ & $\phi(8619)$

15. Write distinct residue classes modulo 7 and write addition and multiplication tables modulo 7.

16. Apply the division algorithm to the following polynomials and find the quotient & remainder.

(i) $f(x) = 2x + 3$, $g(x) = x^2 + 4x + 2$ (ii) $f(x) = x^2 + 4$, $g(x) = 2x^4 + 7x + 1$

(iii) $f(x) = 2x + 1$, $g(x) = 6x^3 + 7x^2 + 6x + 1$

17. State the relation between the roots and coefficients of the given polynomial

$$5x^5 + 4x^4 + 3x^3 + 2x^2 + x = 0.$$

18. Find a polynomial in $\mathbb{Q}[x]$ whose roots are (i) 4, -2, 3; (ii) 7, 9, -4; (iii) -6, 2, 1.

19. Write the first 5 terms of the following sequences $\left(\frac{n+1}{n^2}\right)$, $\left(\frac{(-1)^n}{2n}\right)$, $\sin\left(\frac{n\pi}{2}\right)$

20. Prove that $-(-a) = a$, for any $a \in R$

21. Determine whether the given functions are one-one or not. (i) $f(x) = \frac{4x-5}{3x-7}$; (ii) $f(x) = \frac{9x+5}{5x+3}$

22. Solve $\frac{dy}{dx} = e^{x-3y}$ using the method of separation of variables.

23. Solve $\frac{dy}{dx} = e^{3x-2y} - x^2 e^{-2y}$ using the method of separation of variable.